

PITHAPUR RAJAH'S GOVERNMENT COLLEGE (A), KAKINADA

DEPARTMENT OF MATHEMATICS



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Linear Dependence of Vectors: Let $V(F)$ be a vector space. A finite subset $\{\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\}$ of vectors V is said to be linearly dependent (L.D), if there exist scalars $a_1, a_2, a_3, \dots, a_n \in F$, not all zero such that $a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 + \dots + a_n\alpha_n = 0$.

Linear Independence of Vectors: Let $V(F)$ be a vector space. A finite subset $\{\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\}$ of vectors V is said to be linearly independent (L. I), if every relation of the form,

$$a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 + \dots + a_n\alpha_n = \bar{0}, \quad a_i' \in F, \forall 1 \leq i \leq n$$

$$\Rightarrow a_1 = 0, a_2 = 0, a_3 = 0, \dots, a_n = 0.$$

Theorem: Every superset of linearly dependent (L.D) set of vectors is linearly dependent (L.D).

Theorem: Every non-empty subset of a linearly independent (L.I) set of vectors is linearly independent (L.I).

Theorem: A set of vectors which contains at least one zero vector is linearly dependent (L.D).

Theorem: A single non-zero vector forms a linearly independent (L.I) set.

Note. 1. L.I subset of a vector space does not contain zero vector.

2. $\{0\}$ is a L.D Set.

3. For a homogeneous system of linear equations. The determinant of the coefficient matrix A is zero. Then the given vectors are linearly dependent.

4. Let F be a field with unity 1. Then the vectors $e_1 = (1,0,0, \dots, 0)$, $e_2 = (0,1,0, \dots, 0)$, $e_3 = (0,0,1, \dots, 0)$, ..., $e_n = (0,0,0, \dots, 1)$ are called standard vectors of $F^n(F)$. The above vectors are always form a linearly independent set in the vector space $F^n(F)$.

Solved Problem(s):

1. Show that the vectors $v_1 = (1,3,2)$, $v_2 = (1,-7,-8)$ and $v_3 = (2,1,-1)$ are linearly dependent in $R^3(R)$.

Sol: Let a , b , c be scalars such that

$$av_1 + bv_2 + cv_3 = 0$$

$$\Rightarrow a(1,3,2) + b(1,-7,-8) + c(2,1,-1) = (0,0,0)$$

$$\Rightarrow (a + b + 2c, 3a - 7b + c, 2a - 8b - c) = (0,0,0)$$

$$\Rightarrow a + b + 2c = 0, 3a - 7b + c = 0, 2a - 8b - c = 0$$

By solving the above equations, we get $a = 3$, $b = 1$, $c = -2$. This shows that the given vectors are linearly dependent.

$$\text{(or)} \begin{vmatrix} 1 & 3 & 2 \\ 1 & -7 & -8 \\ 2 & 1 & -1 \end{vmatrix} = 1(7 + 8) - 3(-1 + 16) + 2(1 + 14) = 0$$

1. Show that the vectors $v_1 = (1,2,3)$, $v_2 = (2,5,7)$, $v_3 = (1,3,5)$ are linearly independent in $R^3(R)$.

Sol. Let x, y, z be scalars such that $xv_1 + yv_2 + zv_3 = 0$

$$\Rightarrow x(1,2,3) + y(2,5,7) + z(1,3,5) = (0,0,0)$$

$$\Rightarrow (x + 2y + z, 2x + 5y + 3z, 3x + 7y + 5z) = (0,0,0)$$

$$\Rightarrow x + 2y + z = 0, 2x + 5y + 3z = 0, 3x + 7y + 5z = 0$$

By solving the above equations, we get $x = y = z = 0$. This shows that the given vectors are linearly independent.

(or)

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 1 & 3 & 5 \end{vmatrix} = 1 \neq 0$$

Problem(s)

1. Determine the following subsets of R^3 are linearly independent or linearly dependent.
 1. $\{(-1,2,1), (3,0,-1), (-5,4,3)\}$,
 2. $\{(1,1,2,4), (2,-1,-5,2), (1,-1,-4,0), (2,1,1,6)\}$
2. Show that the vectors $\{(2, -3), (6, -9)\}$ in $R^2(R)$ are linearly dependent.
3. Determine whether the set of vector $\{(1, -2, 1), (2, 1, -1), (7, -4, 1)\}$ is linearly dependent or Linearly independent.

Note:

1. If two vectors are linearly dependent, then one of them is a scalar multiple of other.
2. If the vectors (x_1, y_1) and (x_2, y_2) are linearly dependent, then
$$x_1y_2 - x_2y_1 = 0.$$

The background is a light gray gradient. In the top-left and bottom-right corners, there are several realistic water droplets of various sizes, rendered with highlights and shadows to give them a 3D appearance.

THANK YOU